1. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

$a = 20{,}000 \quad r = 1.08 \quad u_n = ar^{n-1}$

a) Year 3 $= u_3$

$u_3 = 20{,}000 \times (1.08)^2 = £23{,}328$ 1

b) find n such that $u_n > 65{,}000$

$20{,}000(1.08)^{n-1} > 65{,}000$ 1

$(1.08)^{n-1} > \frac{13}{4}$

$n-1 > \frac{\log(13/4)}{\log(1.08)}$ 1

$n > 16.31\ldots$

n is an integer so $n = 17$.

Profits first exceed £65,000 in Year 17 1

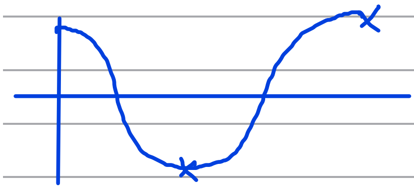
c) $S_{20} = \frac{20{,}000[1-(1.08)^{20}]}{1-1.08}$ 1 $= £915{,}000$ 1

(nearest 1000)

2. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)



$\cos(180n)^\circ = (-1)^n$

so $\displaystyle\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \sum_{n=2}^{\infty} \left(-\frac{3}{4}\right)^n$

$\left(\frac{3}{4}\right)^n (-1)^n = \left(-\frac{3}{4}\right)^n$

geometric series with

$a = \left(-\frac{3}{4}\right)^2 = \frac{9}{16}$ 1

$r = -\frac{3}{4}$

1

$S_\infty = \dfrac{a}{1-r} = \dfrac{\frac{9}{16}}{1+\frac{3}{4}} = \dfrac{9}{28}$ 1

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \xrightarrow{\times r} 5 + 2\sin\theta \quad \text{and} \quad \xrightarrow{\times r} 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

(3)

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k(1 - \sqrt{3})$$

where k is a constant to be found.

(5)

a) $r = \dfrac{5 + 2\sin\theta}{12\cos\theta} = \dfrac{6\tan\theta}{5 + 2\sin\theta}$ (1)

$(5 + 2\sin\theta)^2 = (6\tan\theta)(12\cos\theta)$

$(5 + 2\sin\theta)^2 = 72\dfrac{\sin\theta}{\cancel{\cos\theta}}\cancel{\cos\theta}$ (1)

$25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$

$\therefore\ 4\sin^2\theta - 52\sin\theta + 25 = 0$ (1)

b) $(2\sin\theta - 25)(2\sin\theta - 1) = 0$

$\sin\theta = \frac{25}{2}$ or $\sin\theta = \frac{1}{2}$ (1)

since $-1 \leq \sin\theta \leq 1$,

$\sin\theta = \frac{1}{2}$ only

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ As θ is obtuse, $\theta = \frac{5\pi}{6}$ only (1)

c) $S_\infty = \frac{a}{1-r}$ for $|r| < 1$

$a = 12\cos\theta = 12\cos\frac{5\pi}{6} = -6\sqrt{3}$ (1)

$r = \frac{5 + 2\sin\left(\frac{5\pi}{6}\right)}{12\cos\left(\frac{5\pi}{6}\right)} = \frac{-\sqrt{3}}{3}$ (1)

$\therefore S_\infty = \frac{-6\sqrt{3}}{1-\left(-\frac{\sqrt{3}}{3}\right)}$ (1)

$= \frac{-18\sqrt{3}}{3+\sqrt{3}} \times \left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$ (1) to rationalise the denominator

$= \frac{-54\sqrt{3} + 54}{9-3}$

$= -9\sqrt{3} + 9$

$= 9(1-\sqrt{3})$ (1) $\therefore k = 9$

4. The first three terms of a geometric sequence are

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0$$

(2)

Given that the sequence converges,

(b) (i) find the value of k, giving a reason for your answer,

(ii) find the value of S_∞

(5)

a) $\frac{12-3k}{3k+4} = \frac{k+16}{12-3k}$ 1 ← because scale between two terms should be the same.

$(12-3k)(12-3k) = (k+16)(3k+4)$

$144 - 72k + 9k^2 = 3k^2 + 52k + 64$

$6k^2 - 124k + 80 = 0$

$3k^2 - 62k + 40 = 0$ 1

b) $(k-20)(3k-2) = 0$

$\therefore k = 20$ or $k = \frac{2}{3}$ 1

if $k = 20$, $r = \frac{12-3(20)}{3(20)+4} = -\frac{3}{4}$ $\quad \left|-\frac{3}{4}\right| < 1$ ✓

if $k = \frac{2}{3}$, $r = \frac{12-3\left(\frac{2}{3}\right)}{3\left(\frac{2}{3}\right)+4} = \frac{5}{3}$

$\left|\frac{5}{3}\right| > 1$, so if $k = \frac{2}{3}$ the series cannot converge.

$\therefore k = 20$ 1

$k = 20 \quad \therefore \quad a = 3(20) + 4 = 64$ ①

$r = -\frac{3}{4}$

$$S_\infty = \frac{a}{1-r} = \frac{64}{1+\frac{3}{4}} = \frac{256}{7}$$ ① ①